

A-level maths worksheet

A-level Maths | Pure, statistics and mechanics

Name _____ Class _____ Date _____

Score _____ / 24

A calculator is allowed. Take $g = 9.8 \text{ m/s}^2$ in mechanics questions. Show full working and give non-exact answers to 3 significant figures unless told otherwise.

Example: $\frac{d}{dx} (5x^4 - 3x) = 20x^3 - 3$. Multiply by the power, then reduce the power by one.

Section A · Pure mathematics

1. Differentiate $y = 4x^3 - 5x^2 + 7x - 2$.

2. Find $\int (6x^2 - 4x + 3) \, dx$.

3. Evaluate $\int_1^3 (3x^2 + 2) \, dx$.

4. Solve $\log_2 x = 5$.

5. Solve $e^{2x} = 7$. Give your answer to 3 significant figures.

6. Find the coordinates of the stationary point on $y = x^2 - 6x + 11$.

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7. Find the coefficient of x^3 in the expansion of $(1 + 2x)^6$.

Answer

8. An arithmetic series has first term 5 and common difference 3. Find the sum of the first 20 terms.

Answer

9. A geometric series has first term 8 and common ratio $\frac{1}{2}$. Find its sum to infinity.

Answer

10. Solve $\sin x = 0.5$ for $0^\circ \leq x \leq 360^\circ$.

Answer

11. Differentiate $y = x^2 e^{3x}$.

Answer

12. Find the centre and radius of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$.

Answer

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Section B · Statistics

13. $X \sim B(10, 0.3)$. Find $P(X = 2)$ to 4 decimal places.

Answer

14. $X \sim B(10, 0.3)$. Find $E(X)$ and $\text{Var}(X)$.

Answer

15. $X \sim N(50, 4^2)$. Find $P(X < 56)$ to 4 decimal places.

Answer

16. A and B are independent events with $P(A) = 0.4$ and $P(B) = 0.5$.
Find $P(A \cap B)$.

Answer

17. $P(C) = 0.6$, $P(D) = 0.3$ and $P(C \cup D) = 0.72$. Find $P(C \cap D)$ and
state whether C and D are independent.

Answer

18. For the data 2, 4, 4, 4, 5, 5, 7, 9 find the mean and the standard
deviation (divide by n).

Answer

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Section C · Mechanics

- 19.** A car accelerates uniformly from 4 m/s to 22 m/s in 6 s. Find its acceleration.

Answer

- 20.** For the same car (4 m/s to 22 m/s in 6 s), find the distance travelled.

Answer

- 21.** A ball is dropped from rest and falls 10 m. Taking $g = 9.8 \text{ m/s}^2$, find its speed, to 3 significant figures.

Answer

- 22.** A resultant force acts on a 5 kg particle, giving it an acceleration of 2.4 m/s^2 . Find the force.

Answer

- 23.** A 3 kg box on a rough horizontal floor ($\mu = 0.2$) is pulled by a horizontal force of 15 N. Taking $g = 9.8 \text{ m/s}^2$, find its acceleration.

Answer

- 24.** A particle has velocity $v = 3t^2 - 4t \text{ m/s}$. Find its acceleration when $t = 2$.

Answer

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Check each answer and its method. Equivalent forms are correct unless a question asks for a particular form or degree of accuracy.

Section A · Pure mathematics

1. $\frac{dy}{dx} = 12x^2 - 10x + 7$

Multiply by the power and reduce it by one for each term; the constant differentiates to 0.

2. $2x^3 - 2x^2 + 3x + c$

Raise each power by one and divide by the new power. Include the constant of integration, c.

3. **30**

Integrate to get $x^3 + 2x$. Then $(27 + 6) - (1 + 2) = 33 - 3 = 30$.

4. **$x = 32$**

Rewrite in exponential form: $x = 2^5 = 32$.

5. **$x = 0.973$**

Take natural logs: $2x = \ln 7$, so $x = \frac{1}{2} \ln 7 = 0.97295\dots$

6. **(3, 2)**

$\frac{dy}{dx} = 2x - 6 = 0$ gives $x = 3$. Then $y = 9 - 18 + 11 = 2$.

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7. 160

The x^3 term is $6C3 \times (2x)^3 = 20 \times 8x^3 = 160x^3$.

8. 670

$S_n = n/2 [2a + (n - 1)d] = 10 \times (10 + 57) = 670$.

9. 16

$S_\infty = a/(1 - r) = 8 \div \frac{1}{2} = 16$. This is valid because $|r| < 1$.

10. $x = 30^\circ$ or 150°

$\sin^{-1}(0.5) = 30^\circ$. Sine is also positive in the second quadrant: $180^\circ - 30^\circ = 150^\circ$.

11. $dy/dx = e^{3x}(3x^2 + 2x)$

Product rule with $u = x^2$ and $v = e^{3x}$: $u'v + uv' = 2xe^{3x} + 3x^2e^{3x}$.

12. Centre (3, -2), radius 5

Complete the square: $(x - 3)^2 + (y + 2)^2 = 12 + 9 + 4 = 25$, so $r = \sqrt{25} = 5$.

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Section B · Statistics

13. 0.2335

$$P(X = 2) = {}^{10}C_2 \times 0.3^2 \times 0.7^8 = 45 \times 0.09 \times 0.0576\dots = 0.23347\dots$$

14. $E(X) = 3$, $\text{Var}(X) = 2.1$

$$E(X) = np = 10 \times 0.3 = 3. \text{Var}(X) = np(1 - p) = 3 \times 0.7 = 2.1.$$

15. 0.9332

$$\text{Standardise: } z = (56 - 50) \div 4 = 1.5. \text{ Then } P(Z < 1.5) = 0.93319\dots$$

16. 0.2

$$\text{For independent events, } P(A \cap B) = P(A) \times P(B) = 0.4 \times 0.5 = 0.2.$$

17. $P(C \cap D) = 0.18$; independent

$$P(C \cap D) = 0.6 + 0.3 - 0.72 = 0.18. \text{ Since } P(C) \times P(D) = 0.18 \text{ too, } C \text{ and } D \text{ are independent.}$$

18. Mean 5, standard deviation 2

$$\text{Mean} = 40 \div 8 = 5. \Sigma(x - 5)^2 = 32, \text{ so the variance is } 32 \div 8 = 4 \text{ and the s.d. is } \sqrt{4} = 2.$$

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Section C · Mechanics

19. 3 m/s²

$v = u + at$, so $22 = 4 + 6a$, giving $a = 18 \div 6 = 3 \text{ m/s}^2$.

20. 78 m

$s = \frac{1}{2}(u + v)t = \frac{1}{2} \times (4 + 22) \times 6 = 78 \text{ m}$.

21. 14.0 m/s

$v^2 = u^2 + 2as = 0 + 2 \times 9.8 \times 10 = 196$, so $v = 14.0 \text{ m/s}$.

22. 12 N

Newton's second law: $F = ma = 5 \times 2.4 = 12 \text{ N}$.

23. 3.04 m/s²

$R = 3 \times 9.8 = 29.4 \text{ N}$, so friction $= 0.2 \times 29.4 = 5.88 \text{ N}$. Then $a = (15 - 5.88) \div 3 = 3.04 \text{ m/s}^2$.

24. 8 m/s²

$a = dv/dt = 6t - 4$. At $t = 2$: $a = 12 - 4 = 8 \text{ m/s}^2$.